# Solving A Discrete Challenge: The Farmer's Daughter Problem

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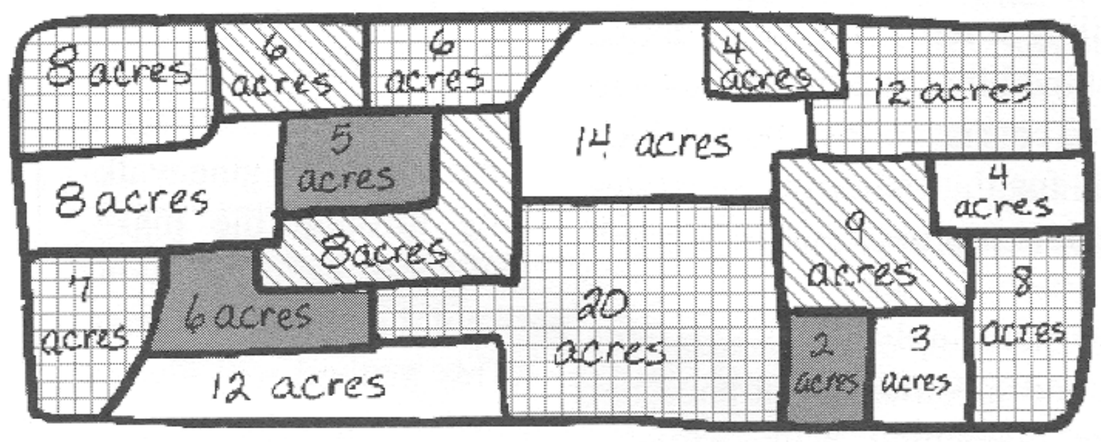
In the last issue of *In Discrete Mathematics, The Farmer's Daughter,* an interesting map coloring problem written by Jill Dunlap was presented. In this problem, a farmer had divided his small farm into 18 different fields for each of which he wished to purchase livestock. Although the farmer enjoyed many different animals, he wanted to limit his animals to as few types as possible. He also knew that he could not have like animals in pastures that share a common border because the animals would try to tear down the fences to be with their friends in the adjoining fields.




The farmer presented his situation to his problem-solving daughter with the following guidelines:

1. He wanted as many acres of cows as possible
2. Then as many acres of sheep as possible
3. Then as many acres of horses as possible
4. Then as many acres of goats as possible
5. Finally, as many acres of pigs as possible.

He decided that each acre could hold either 8 cows, 20 sheep, 6 horses, 12 goats, or 18 pigs. It was possible that he could actually end up with more sheep than cows. This was fine, as long as the cows occupied the most acreage. He handed his daughter a diagram of the field (shown below, but without the coloring) and set off with his sons to build fences while she devised a plan for placing the livestock.

A Discrete Challenge was offered to readers and their students to submit their best solutions to this problem. The response was fantastic. We received 51 solutions from students ranging from grade 5 to the college level.

The best solution for the farmer's problem showed that he should place 488 cows in 61 acres, 820 sheep in 41 acres, 162 horses in 27 acres, and 156 goats in 13 acres. He should not buy any pigs. The coloring shown here was submitted by Priya Parayanthal, a 5th-grade student in New Jersey: The cross-hatched areas represent cows, the diagonal regions represent horses, the shaded regions represent goats, and the blank regions represent sheep.

Most students explained that they approached this problem using a combination of guess and check and logical thinking. Students began by placing cows in the 20 acre field because this was the area with the greatest acreage. They then proceeded to place cows in the next largest fields that were not adjacent to the 20 acre sector. This procedure was repeated for the sheep, horses, and goats. After solving the problem in this manner, many students confirmed their solution by trying to place the cows without utilizing the 20 acre field. They found that this resulted in a lower acreage of cows.

Other discrete math techniques were also used to solve this problem. Some students used map coloring strategies to first find that four types of animals would be needed. Students with knowledge of the Four Color Theorem might have realized that the farmer would not need to buy pigs because, no matter how the farm was partitioned, at most four types of animals would be required to ensure that like animals were not in adjacent fields. It is impossible to use only three animals because there are sectors of the map that are surrounded by an odd number of fields, forcing the use of four animals. A few students modeled the diagram of the farm using a graph, with the vertices representing the different fields and the edges indicating fields that share a

Sixteen students submitted correct solutions to this problem. Unfortunately, due to this large number, we are unable to print the work of all of these students. We would like to congratulate the following people for correctly solving *The Farmer's Daughter*:

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| *Yong S. Cohen* | *(New York, NY)* |
| *Allie DeGeorge* | *(Mrs. Burnett, Patrick McGaheran School, Lebanon, NJ)* |
| *David Dreifus* | *(Mrs. Burnett, Patrick McGaheran School, Lebanon, NJ)* |
| *George Flannery* | *(Mrs. Burnett, Patrick McGaheran School, Lebanon, NJ)* |
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